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Thermocapillary motion of a droplet heated by radiation

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Abstract—A model problem on the steady thermocapillary motion of a droplet suspended in a transparent liquid medium under radiation in the form of a ray, which by absorbing inside the droplet induces its non-uniform heating, is considered. Assuming low Reynolds and Peclet numbers, expressions for the force on the droplet, and the migration velocity of the droplet are derived. The principal results of the work are represented in equations (22) and (23). They are shown to be qualitatively different from that in the case examined in earlier works where radiation was suggested to be entirely absorbed on the droplet surface. The possibility of instability and multiplicity of steady regimes of the motion is pointed out.

1. INTRODUCTION

A VARIATION of interfacial tension along a droplet surface (also along any liquid–liquid interface of course) through stress balance at the surface can essentially influence the motion of fluids on either side of the surface. Investigation of such capillary effects has already found application, for example, in chemical engineering, but it is especially important for enlarging our understanding of the processes occurring in the microgravity environment on board spacecraft when capillarity is often the only mechanism governing the motion. Since interfacial tension usually depends on the temperature and concentration of admixtures, etc., capillary effects can be caused by the gradient in any one of these entities.

Starting from Young *et al.* [1], quite a number of papers have been devoted to the steady thermocapillary motion of a droplet in an unbounded liquid medium due to a constant temperature gradient applied at infinity (for example, see refs. [2–4], with a vast list of references contained in ref. [4]). But this is not the only means of inducing the thermocapillary motion of a droplet. Another possibility, subjecting a droplet to radiation, was pointed out in refs. [5, 6]. Because of the absorption of radiation, a non-uniform temperature field can be created resulting in thermocapillarity.

In refs. [5, 6] a droplet was supposed to be a black or grey body, while the external liquid was absolutely transparent. In other words, radiation was suggested to be entirely absorbed on the surface of the droplet. The other case is considered in the present work, when a droplet slightly absorbs radiation in a volume and as a result there is some distribution of heat generation inside the droplet. As will be seen shortly, the results obtained here are qualitatively different from that of refs. [5, 6].

2. FORMULATION

Consider a droplet illuminated by a radiation beam and suspended in another liquid medium which is much more transparent than a droplet one. In this situation, due to radiation absorption, one pole of the droplet is hotter than the other. Thus, the thermocapillary effect becomes apparent. In the absence of gravity, when thermocapillarity proves to be the only force, the effect entirely determines the motion, which is purely thermocapillary in this case.

The aim of the present work is to evaluate the force on the droplet under radiation due to thermocapillarity and, knowing it, to calculate the droplet velocity in a general case, including the presence of gravity.

The assumptions and idealizations to deal analytically with the problem, and thus, to describe the motion of the droplet are as follows.

The liquids are viscous, unmixable, and incompressible. All their physical properties such as the dynamic viscosities μ_i, the densities ρ_i, the thermal conductivities λ_i, and the thermal diffusivities χ_i are independent from temperature, except for the interfacial tension σ which is linear with temperature T:

$$\sigma = \sigma_0 + (\mathrm{d}\sigma/\mathrm{d}T)(T - T_0), \quad \mathrm{d}\sigma/\mathrm{d}T = \text{const.}$$

σ_0 and T_0 are some fixed values. Here and afterwards the subscripts $i = 1, 2$ denote the external and internal fluids, respectively. The interfacial tension is large enough to preserve the spherical shape of the droplet.

The simplest possible assumptions are made for optics of the process, claiming that: (1) radiation of the flux J is in the form of a uniform parallel ray, the droplet being entirely inside the ray; (2) there is neither reflection, nor refraction on the surface of the droplet; (3) the external liquid is transparent, while the internal one absorbs radiation according to

NOMENCLATURE

A coefficient
A_n ($n = 2, 3, \ldots$) coefficients, equations (12) and (13)
$\mathbf{B}$ buoyancy force on the droplet
E^2 differential operator
$\mathbf{e}$ unit vector in the radiation propagation direction
$\mathbf{F}, F$ hydrodynamical force on the droplet and its dimensionless component in the radiation propagation direction
$f_{in}(r)$ functions, equations (17) and (18)
$\mathbf{G}$ gravity force on the droplet
$G_n(\mu)$ ($n = 2, 3, \ldots$) Gegenbauer polynomial of order n and degree $-1/2$
J radiation flux
K_{in} ($n = 1, 2, \ldots$) coefficients, equations (17) and (18)
Ma modified Marangoni number
m parameter combination defined in equation (20)
m_n ($n = 1, 2, \ldots$) critical parameter combinations
$P_n(\mu)$ ($n = 1, 2, \ldots$) Legendre polynomial of order n
q intensity of internal heat generation per unit volume
r radial coordinate measured from the droplet centre [dimensionless]
T_i dimensionless temperature field
$\mathbf{U}$ droplet migration velocity
$\mathbf{U}_\infty, U_\infty$ fluid velocity at infinity in a reference frame travelling with the droplet and its dimensionless component
$\mathbf{U}_{\lim}$ limit value of the droplet migration velocity when radiation flux tends to infinity
$v_{ir}, v_{i\theta}$ components of a velocity field in the r- and θ-directions
z axis passing through the droplet centre in the radiation propagation direction.

Greek symbols
α radiation absorption coefficient of the droplet phase
β ratio of the dynamic viscosities, μ_2/μ_1
δ ratio of the thermal conductivities, λ_2/λ_1
ε small parameter, αa
κ ratio of the thermal diffusivities, χ_2/χ_1
λ_i thermal conductivity
μ $\cos\theta$
μ_i dynamic viscosity
σ surface tension
χ_i thermal diffusivity
ψ_i stream function [dimensionless].

Subscripts
i 1, continuous phase; 2, droplet phase
∞ evaluation at large distance from the droplet.

Buger's law; (4) a correlation $\alpha a \ll 1$ holds, where a is the droplet radius and α the absorption coefficient. The latter means that the absorption is weak, so it is possible to approximately replace Buger's exponent by a linear function. Thus, the function $q(r, \theta)$ of heat generation intensity per unit volume inside the droplet can be written as

$$q(r, \theta) = \alpha J[1 - \alpha(r\cos\theta + \sqrt{(a^2 - r^2\sin^2\theta)})] \quad (1)$$

where r is the radial coordinate measured from the droplet centre; θ the polar angle measured from the positive direction of the z-axis which is chosen to pass through the droplet centre and to point in the direction the radiation propagates in.

The solution is constructed at low Reynolds and Peclet numbers and only a zero approximation in Reynolds number, or creeping flow approximation, is employed. The velocity the droplet migrates at and the body force, if any, are supposed to be parallel to the direction the radiation propagates in (later on this requirement will be rejected). Thus, the problem can be treated as axisymmetrical, with the z-axis being the symmetrical one. The consideration is held in a reference frame travelling with the droplet centre, in terms of the stream function ψ_i defined by

$$v_{ir} = \frac{1}{r^2\sin\theta}\frac{\partial\psi_i}{\partial\theta}, \quad v_{i\theta} = -\frac{1}{r\sin\theta}\frac{\partial\psi_i}{\partial r}$$

where v_{ir}, $v_{i\theta}$ are components of the velocity field in the r- and θ-directions.

As stated earlier, the main task is to derive the hydrodynamical force (usually known as a drag) on the droplet as a function of all the parameters involved, the migration velocity among them. Then, by setting up the net force on the droplet, which can be, for example, superposition of the hydrodynamical force and the mass force, if any, to equal zero, the migration velocity can be obtained.

Let us proceed with mathematical formulation of the problem. First of all, dimensionless variables are to be introduced. A natural length scale is the droplet radius a. The temperature T_i is non-dimensionalized by subtracting its constant value at infinity and dividing by the scale $\alpha J a^2/\lambda_1$. The quantity $(\mathrm{d}\sigma/\mathrm{d}T)a^2 J a^3/(u_i\lambda_i)$ is used to define the scaled velocity field. Finally, the equations and boundary conditions for the dimensionless stream function and temperature fields can be written as

$$E^4\psi_i = 0, \quad E^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\mu^2}{r^2}\frac{\partial^2}{\partial \mu^2},$$

$$E^4 = E^2(E^2) \tag{2}$$

$$r \to \infty, \quad \psi_1 \to U_\infty r^2(1-\mu^2)/2;$$

$$r \to 0, \quad \psi_2/r^2 < \infty \tag{3}$$

$$\mu = \pm 1, \quad \psi_i = 0, \quad E^2\psi_i = 0 \tag{4}$$

$$r = 1, \quad \psi_1 = \psi_2 = 0, \quad \partial\psi_1/\partial r = \partial\psi_2/\partial r \tag{5}$$

$$\left(2\frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2}\right)(\psi_1 - \beta\psi_2) = \frac{1}{\varepsilon}(1-\mu^2)\frac{\partial T_1}{\partial \mu} \tag{6}$$

$$\frac{Ma*\varepsilon}{r^2}\frac{\partial(\psi_1, T_1)}{\partial(r,\mu)} = \Delta T_1 \tag{7}$$

$$\frac{\kappa^{-1}Ma*\varepsilon}{r^2}\frac{\partial(\psi_2, T_2)}{\partial(r,\mu)} = \Delta T_2$$
$$+\delta^{-1}[1-\varepsilon(r\mu+\sqrt{(1-r^2+r^2\mu^2)})] \tag{8}$$

$$r \to \infty, \quad T_1 \to 0; \quad r \to 0, \quad T_2 < \infty \tag{9}$$

$$\mu \to \pm 1, \quad (1-\mu^2)\partial T_i/\partial\mu \to 0 \tag{10}$$

$$r = 1, \quad T_1 = T_2, \quad \partial T_1/\partial r = \delta\partial T_2/\partial r$$

$$\beta = \frac{\mu_2}{\mu_1}; \quad \delta = \frac{\lambda_2}{\lambda_1}; \quad \kappa = \frac{\chi_2}{\chi_1};$$

$$Ma = \frac{d\sigma}{dT}\frac{\alpha Ja^3}{\mu_1\lambda_1\chi_1}$$

$$i = 1,2; \quad \mu = \cos\theta; \quad \varepsilon = \alpha a. \tag{11}$$

The modified Marangoni number Ma has been introduced here. Conventional symbols have been used for the differential operations. The quantity U_∞ is the scaled velocity of the uniform flow at infinity. Equations (2) are the Stokes equations in terms of stream function, while equations (7) and (8) are the energy equations. The function on the right-hand side of equation (8) in the form (1) is attributed to the heat generation inside the droplet due to the radiation absorption. Conditions (3) and (9) determine the behaviour of the velocity and temperature fields at infinity and in the centre of the droplet. The absence of peculiarities at the symmetry axis is indicated in equations (4) and (10). Boundary conditions (5) are responsible for the vanishing of the normal velocity and continuity of the tangential one at the droplet surface, while a continuity of the temperature and thermal flux in the same place is reflected in equation (11). The tangential stress balance concerning viscous and capillary stresses can be written as equation (6).

The system (2)–(11) does not contain a normal stress balance condition since it can be replaced by a more convenient condition of a net force on the droplet vanishing to zero, when a departure of the droplet shape from a spherical one is neglected. The unknown quantity U_∞ is to be determined from this condition as stated earlier.

3. SOLUTION

The formal solution of the problem (2)–(4) may be written on the grounds of the general solution contained in the book by Happel and Brenner [7] as follows:

$$\psi_1 = U_\infty\left(r^2 - \frac{1}{r}\right)G_2(\mu) + \sum_{n=2}^{\infty} A_n(r^{-n+3} - r^{-n+1})G_n(\mu) \tag{12}$$

$$\psi_2 = \frac{3}{2}U_\infty(r^4 - r^2)G_2(\mu) + \sum_{n=2}^{\infty} A_n(r^{n+2} - r^n)G_n(\mu). \tag{13}$$

Here, $G_n(\mu)$ is the Gegenbauer polynomial of order n and degree $-1/2$. The unknown constants in equations (12) and (13) are to be determined from equation (6) when solving the temperature problem.

Knowing equation (12), the expression for the hydrodynamical force on the droplet, scaled by $(d\sigma/dT)\alpha^2 Ja^4/\lambda_1$, may be written as [7]

$$F = -4\pi A_2. \tag{14}$$

Note that the quantities U_∞ and F are components of the corresponding vectors in the z-axis direction and can be both positive and negative (the other components are zero).

As assumed, the quantity ε is small. This fact has been already used to represent the heat generation term on the right-hand side of equation (8). Now we shall develop an asymptotic expansion for the temperature field in the limit $\varepsilon \to 0$, the other dimensionless numbers being of order unity. In this limit, the number $Ma*\varepsilon$ here representing a Peclet number, is also small to agree with the suggestions involved.

To find an asymptotic solution for the problem (6)–(13), the method of matched asymptotic expansions is to be employed. The procedure to be accomplished is well known and straightforward (e.g. see ref. [8]), only slight complication due to the unknown constants in the infinite series (equations (12) and (13)) to be found emerges here, so the mediate details are omitted, with the final result up to $O(\varepsilon)$ for the temperature field being represented as follows.

In the outer region ($r > O(\varepsilon^{-1})$)

$$T_1 = T_1^{(0)} + \varepsilon T_1^{(1)}.$$

In the inner region ($1 < r < O(\varepsilon^{-1})$)

$$T_1 = T_{10} + \varepsilon T_{11}.$$

Inside the droplet

$$T_2 = T_{20} + \varepsilon T_{21}. \tag{15}$$

Here

$$T_1^{(0)} = 0, \quad T_{10} = \frac{1}{3r}, \quad T_{20} = \frac{1}{3}\left[1 + \frac{1}{2\delta}(1-r^2)\right]$$

$$\varepsilon T_1^{(1)} = \frac{1}{3r}\exp\{\tfrac{1}{2}U_\infty\, Ma\,\varepsilon r[\mu - \text{sign}\,(U_\infty\, Ma)]\} \quad (16)$$

$$T_{11} = -\frac{1}{6}U_\infty\, Ma\,\text{sign}\,(U_\infty\, Ma)$$
$$+\frac{1}{3}U_\infty\, Ma\left\{\frac{1}{2}\left(1+\frac{1}{2r^3}\right)\right.$$
$$\left.-\frac{1}{\delta+2}\left[\frac{3}{4}(\delta+1)+\frac{3}{35}\kappa^{-1}\right]\frac{1}{r^2}\right\}P_1(\mu)$$
$$+\frac{1}{6}Ma\sum_{n=1}^{\infty}\left[K_{1n}\frac{1}{r^{n+1}}+\frac{1}{(n+1)r^{n+2}}\right.$$
$$\left.+\frac{1}{nr^n}\right]A_{n+1}P_n(\mu)+\sum_{n=0}^{\infty}f_{1n}(r)P_n(\mu)$$

$$T_{21} = -\frac{1}{6}U_\infty\, Ma\,\text{sign}\,(U_\infty\, Ma)$$
$$+\frac{1}{3}U_\infty\, Ma\left\{\frac{1}{2+\delta}\left[\frac{3}{4}-3\kappa^{-1}\frac{18\delta^{-1}+17}{280}\right]r\right.$$
$$\left.+\frac{3}{2}\kappa^{-1}\delta^{-1}\left[\frac{1}{10}r^3-\frac{1}{28}r^5\right]\right\}P_1(\mu)$$
$$+\frac{1}{6}Ma\sum_{n=1}^{\infty}\left[K_{2n}r^n-\delta^{-1}\kappa^{-1}\left(\frac{r^{n+4}}{4n+10}\right.\right.$$
$$\left.\left.-\frac{r^{n+2}}{2n+3}\right)\right]A_{n+1}P_n(\mu)+\sum_{n=0}^{\infty}f_{2n}(r)P_n(\mu) \quad (17)$$

$$K_{1n} = -(1+n+n\delta)^{-1}\left[\frac{3+2n+\delta(2n+1)}{n+1}\right.$$
$$\left.+4\kappa^{-1}\frac{1}{(2n+3)(2n+5)}\right]$$

$$K_{2n} = (1+n+n\delta)^{-1}\left[\frac{1}{n(n+1)}\right.$$
$$\left.+\kappa^{-1}\frac{n+4+\delta^{-1}(n+1)}{4n+10}-\kappa^{-1}\frac{n+2+\delta^{-1}(n+1)}{2n+3}\right]$$

$$f_{11}(r) = -\frac{1}{5(2+\delta)r^2}$$

$$f_{21}(r) = \left(\frac{1}{10}r^3-\frac{2+3\delta}{10(2+\delta)}r\right)\delta^{-1} \quad (18)$$

$$\text{sign}\,(x) = \begin{cases} 1, & \text{if}\, x>0 \\ -1, & \text{if}\, x<0. \end{cases}$$

Here $P_n(\mu)$ is the Legendre polynomial of order n. Condition (6) has not yet been employed, so the unknown constants U_∞, A_n $(n = 2, 3, \ldots)$ remain. Strictly, these constants are also to be expanded into a series in ε and it is the order unity ground terms of these expansions that are meant in equations (17) and (18). The terms, in which the functions $f_{in}(r)$ $(n = 0, 1, \ldots)$ occur, are due to the non-homogeneous term on the right-hand side of equation (8) which is expanded into Legendre polynomials when solving the problem. Expressions for these functions are not supplied (nevertheless, it is obvious that the odd number functions beginning with the third one equal zero). The thing is that, as can be seen from equation (14) only one mode contributes to the force, and thus, to the migration velocity of the droplet which is the main object of interest in this work. So one may consider only that mode. Still, absolutely ignoring the higher modes is not quite relevant, since such phenomena as peculiarity and instability of the solution have been proved to occur in these modes. As will be seen further, from the viewpoint of these phenomena, the information represented in equations (17) and (18) is quite sufficient.

Substituting equations (12), (13), (15)–(17) into equation (6), taking into account the formula $(1-\mu^2)\,\mathrm{d}P_n/\mathrm{d}\mu = n(n+1)G_{n+1}$, and solving for the G_2-mode, one can obtain

$$A_2 = -\left\{\left[1+\frac{3}{2}\beta+m\left(\frac{3}{4}-\frac{3}{35}\kappa^{-1}\right)\right]U_\infty\right.$$
$$\left.+\frac{1}{15(2+\delta)}\right\}*\left[1+\beta+m\left(\frac{1}{4}-\frac{2}{35}\kappa^{-1}\right)\right]^{-1} \quad (19)$$

$$m = -\frac{Ma}{9(2+\delta)}. \quad (20)$$

Solving for the higher modes $(n = 3, 4, \ldots)$ we need only one interesting detail, namely, the existence of such combinations of the parameters involved, at which the equations for determination of the constants A_n reduce to $0 * A_n = 0$ for even numbers n and to $0 * A_n = \text{const.} \neq 0$ for odd numbers. The combinations can be presented as $m = m_n$, where

$$m_n = \frac{(2n-1)(n+n\delta-\delta)(1+\beta)}{3(2+\delta)}$$
$$\times\left[\kappa^{-1}\frac{n(n-1)}{(2n+1)(2n+3)}-\frac{1}{4}\right]^{-1}. \quad (21)$$

We shall return to this fact in the next section. For $m \neq m_n$ we have a single finite solution for A_n $(n = 3, 4, \ldots)$.

Substitution of equation (19) into equation (14) gives the dimensionless expression for the hydrodynamical force. After returning to variables with dimensions, rewriting in a vector form, and introducing the droplet migration velocity $\mathbf{U}(\mathbf{U} = -\mathbf{U}_\infty)$, this expression may be written as

$$F = 4\pi\mu_1 a\left\{-\left[1+\frac{3}{2}\beta+m\left(\frac{3}{4}-\frac{3}{35}\kappa^{-1}\right)\right]\mathbf{U} + \frac{d\sigma}{dT}\frac{\alpha^2 Ja^3}{15\mu_1\lambda_1(2+\delta)}\mathbf{e}\right\} * \left[1+\beta+m\left(\frac{1}{4}-\frac{2}{35}\kappa^{-1}\right)\right]^{-1} \quad (22)$$

where $\mathbf{e}$ is the unit vector in the radiation propagation direction.

Note that despite formula (22) being derived when the vectors $\mathbf{U}$ and $\mathbf{e}$ are collinear, it holds even when this condition is infringed. This is due to linearity of the hydrodynamical problem involved, so different factors may be considered separately, the results added together.

With the help of equation (22) the droplet migration velocity can be easily obtained by setting the net force on the droplet equal to zero. With regard to the Earth's gravity this condition reduces to $\mathbf{F}+\mathbf{G}+\mathbf{B} = 0$. Here $\mathbf{G}$ is the gravity force and $\mathbf{B}$ the buoyancy force. As stated earlier, in the general case, it is not necessary for the vector $\mathbf{e}$ (and, consequently, the vector $\mathbf{U}$) to be collinear to $\mathbf{G}$ and $\mathbf{B}$. In particular, the expression for the migration velocity in a free fall environment ($\mathbf{G} = \mathbf{B} = 0$) is as follows:

$$\mathbf{U} = \frac{d\sigma}{dT}\frac{\alpha^2 Ja^3}{15\mu_1\lambda_1(2+\delta)}\left[1+\frac{3}{2}\beta+m\left(\frac{3}{4}-\frac{3}{35}\kappa^{-1}\right)\right]^{-1}\mathbf{e}. \quad (23)$$

Formulae (22) and (23) represent the principal results of the present work.

4. ANALYSIS

As can be seen from equation (23), the dependence of the migration velocity of the droplet on the radiation flux is non-linear and so more complex than the linear one obtained in refs. [5, 6] (note that $m \sim J$). This dependence is shown qualitatively in Fig. 1 for different relationships between the parameters. At $\kappa^{-1} = 35/4$, degeneration takes place when the results obtained here are qualitatively similar to that of refs. [5, 6] from the viewpoint of $\mathbf{U}$ against J. The remarkable fact is for the migration velocity to approach the finite value ($\kappa^{-1} \neq 35/4$):

$$\mathbf{U}_{\lim} = -\frac{3\chi_1\alpha}{5}\left(\frac{3}{4}-\frac{3}{35}\kappa^{-1}\right)^{-1}\mathbf{e} \quad (24)$$

when the flux J tends to infinity. This was not quite evident beforehand. So one of the conclusions made on the grounds of the present work is that it is important, if the radiation is absorbed in the volume or on the surface of the droplet. Formula (24), where the minimum number of the parameters is involved, is a good replacement of equation (23) when

$$|[3/4-(3/35)\kappa^{-1}]m| \gg |1+(3/2)\beta|. \quad (25)$$

Some peculiarity is contained in equation (23). When $m = m_1$, where

$$m_1 = [1+(3/2)\beta][(3/35)\kappa^{-1}-3/4]^{-1}$$

the migration velocity of the droplet takes an infinite value. Thus, the results obtained here are incorrect for the values of the parameters m in some vicinity of the value m_1 when applied to a real situation at finitely small Reynolds and Peclet numbers rather than at the asymptotic limit of these numbers. The same may be stated for the vicinities of the values m_n from equation (21) for odd $n = 3, 5, \ldots$ here, but in general the non-model case, may be for all $n = 3, 4, \ldots$.

Expression (22) contains one more peculiarity. At $m = m_2$, where $m_2 = (1+\beta)[(2/35)\kappa^{-1}-1/4]^{-1}$, the denominator in equation (22) becomes zero. But this peculiarity does not become apparent if the numerator in equation (22) also equals zero, i.e. equation (23) holds. It is interesting that at $m = m_2$ expression (23) determines the migration velocity not only in a free fall environment, but also under gravity, the velocity being independent from gravity (at least under the approximation involved). The peculiarity at $m = m_2$

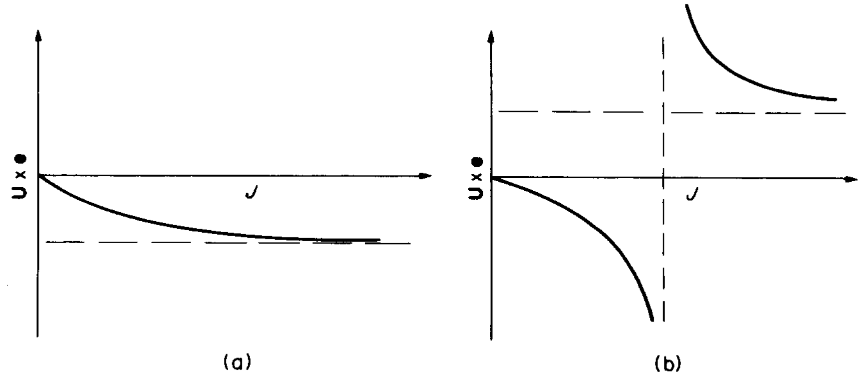

FIG. 1. Qualitative dependence of the droplet migration velocity component in the radiation propagation direction on the radiation flux: (a) for $d\sigma/dT < 0$, $3/4-(3/35)\kappa^{-1} > 0$; (b) for $d\sigma/dT < 0$, $3/4-(3/35)\kappa^{-1} < 0$.

becomes apparent only at such statements of the problem when the droplet velocity different from equation (23) is given before the force needed to support this motion is sought. In the last case, when a real situation is concerned, the consideration is incorrect for the values of the parameter m in some vicinity of the value m_2. Closer to equation (23) is the droplet velocity, the shorter is the incorrect interval of m about the value m_2. Note that the value m_2 can be obtained from equation (21) at $n = 2$.

When the value m is rather far from the critical values m_1, m_n (odd $n = 3, 5, \ldots$), Reynolds and Peclet numbers for the droplet migration at the velocity (equation (23)) can be approximately superevaluated by the velocity scale $\chi_1\alpha$ given by equation (24). Herewith, if the thermal diffusivities and kinematic viscosities are approximately of the same order, it is clear that the assumption of slight radiation absorption inside the droplet ($\varepsilon \ll 1$) automatically results in a validity of the low Reynolds and Peclet numbers assumption for the motion regime found above.

Is the migration of the droplet under the radiation stable? An attentive look at expression (22) from this standpoint induces some doubts. Indeed, the quantity

$$A = -\left[1+\frac{3}{2}\beta+m\left(\frac{3}{4}-\frac{3}{35}\kappa^{-1}\right)\right] * \left[1+\beta+m\left(\frac{1}{4}-\frac{2}{35}\kappa^{-1}\right)\right]^{-1} \quad (26)$$

(to distinguish, the letter A is used here without a subscript) can be both positive and negative depending on the parameter values. If it is positive, the following simple qualitative consideration discloses the instability. A slight departure of the migration velocity of the droplet from the equilibrium value (equation (23)) changes the force on the droplet (equation (22)) to promote further growth of this departure. But it is necessary to keep in mind that this speculation is in no way a rigorous proof and serves only as a qualitative indication to the instability. The quantity A changes its sign when the parameter m passes through the critical values m_1, m_2. The detailed investigation of this question lies beyond the scope of this paper. To add, instability can occur not only in the G_2-mode connected with the droplet migration, but also in the higher modes. The authors believe that while the parameter m passes through the values m_n ($n = 3, 4, \ldots$) from equation (21), the G_n-mode changes its state of stability–instability. Note that in the problem under consideration both peculiarities and instability are connected with the critical values m_n ($n = 1, 2, \ldots$).

For experienced readers the following remark is supplied. The critical combinations m_n ($n = 1, 2, \ldots$) can be also obtained from the neutral stability analysis, confirming the belief that the instability discussed above really exists. Some complications in such analysis occur only in the G_n mode because of two critical values available. But the details are omitted here.

The instability suggests that the motion regime found in the present work is not single and there are a number of ones which failed to be derived at the given approximation. At least, considering the next order approximations in low Reynolds and Peclet numbers in the vicinity of the critical values m_n ($n = 1, 2, \ldots$) for the parameter m, herewith changing the velocity scale, one can, firstly, eliminate from the peculiarities, and secondly, reveal steady regimes of the flow. The example of such a consideration, but in the different situation, and only about the value m_1, is presented in ref. [9]. Since analogous analysis enjoys only a small range of applicability and is to be rather extensive, it has not been held here.

By setting

$$\alpha \to 0, \quad \alpha J = q \quad (27)$$

expression (22) reduces to the principal result of the work by the present authors [10], where a droplet with uniform inside heat generation of the intensity q per unit volume was considered, no matter what that heat generation was induced by. The inducing factors can be a chemical reaction, radioactive decay or absorption of radiation in the limit (27).

The qualitative character of the results obtained here is mainly due to the large uniform component of heat generation inside the droplet in comparison with the non-uniform one. Evidently, under uniform inside heat generation only, no interfacial tension gradient occurs on the surface until fluids inside and outside the droplet are at rest. Thus, thermocapillary effects due to such factors occur only once the motion has been already induced by some mechanism, for instance, by the droplet translation under gravity [11] or, as in the present work, by thermocapillary tensions due to non-symmetry in heat generation; in addition, the thermocapillary flow can occur autonomously, without any inducing circumstances [10]. Note that the results (equations (22), (23)) can be obtained from the rather general expressions for the capillary force on a droplet with uniform inside heat generation and for its migration velocity, contained in ref. [10], but the authors preferred to deduce everything from the very beginning here. Investigation of symmetrical factors capable of creating capillary effects in the process of motion was started in ref. [12], when a droplet with chemical reaction on the surface was considered, and then continued in refs. [9, 13, 14] also for surface chemical reaction, and in ref. [11] for uniform inside heat generation, the autonomous capillary motion being considered in refs. [9, 10, 13].

5. CONCLUDING REMARKS

The investigation undertaken here deals with a model problem at low Reynolds and Peclet numbers. Nevertheless, it reveals some regularities such as a complex dependence of the migration velocity on the radiation flux, peculiarities, instability and multiplicity of the regimes of the motion which all are surely characteristic of the capillary effects with more complicated and realistic axisymmetrical (and not only axisymmetrical) distribution of heat generation inside the droplet, provided its uniform component (it is to be calculated expanding the function of heat generation into a series of Legendre polynomials) is much larger than the non-uniform one as it takes place in the present development. As for the cases in which the last condition is violated one can claim nothing for sure on the grounds of the above exploration. At least, the analysis based on the ground approximation in low Reynolds and Peclet numbers held in refs. [5, 6] allows the disclosure of no such regularities.

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MOUVEMENT THERMOCAPILLAIRE D'UNE GOUTTE CHAUFFE PAR RADIATION

Résumé—On considérés un problème modèle sur mouvement stationnaire thermocapillaire d'une goutte dans un liquide transparent sous l'effect de radiation en forme d'un rayon qui engendre chauffe hétérogène à l'intérieur de la goutte par absorption du radiation. Supposé que nombres de Reynolds et Peclet sont petit, les expressions pour la force hydrodynamique sur la goutte et pour la vélocité migratoire de la goutte sont deduit. Ces resultates principal du travail sont representé à (22), (23). On montres que ils sont différent qualitativement du tels en le cas examiné en les travaux précédent où il y a fait le supposition que radiation a été absorbé entièrement sur la surface de la goutte. La possibilité pour instabilité et pluralité du regimes stationnaire du mouvement est indiqué.

THERMOKAPILLAREN BEWEGUNG VON TROPFEN DIE MITTELS DER STRALUNG GEWÄRMT WERDEN

Zusammenfassung—Ein Modellproblem im Bereich der stationären thermocapillaren Bewegung von Tropfen, die in einer transparenten Flüssigkeit gelöst sind und bestrahlt werden in der Art, das durch die Absorbtion der Strahlung ungleichformige Erwärmung im Tropfen induziert wird, wird betrachtet. Unter der Annahme winziger Reynoldszahlen und Peclet-zahl, Ausdrücke fur die auf den Tropfen, wirkenden kräfte sowie die Wanderungsgeschwindigkeit des Tropfens werd werden abgeleitet. Die Ergebnisse dieser Arbeit sind in (22) und (23) dargestellt. Diese Ergebnisse sind qualitativ unterscheidlich von denen frühers Arbeiten, bei denen die Strahlung vollständig von der Tropfen—Oberfläche absorbiert wurde. Die Möglichkeit für Instabilitäten und Mehrzahllösungen stationärer Zustände der Bewegung wird evläutert.

ТЕРМОКАПИЛЛЯРНОЕ ДВИЖЕНИЕ КАПЛИ, НАГРЕВАЕМОЙ ИЗЛУЧЕНИЕМ

Аннотация—Исследуется модельная задача равномерного термокапиллярного движения капли в прозрачной жидкой среде при наличии излучения в виде луча, который при поглощении в капле вызывает ее неоднородный нагрев. Выведены выражения для силы, действующей на каплю, и скорости ее миграции в предпожении низких чисел Рейнольдса и Пекле. Основные результаты работы представлены уравнениями (22) и (23). Показано, что они качественно отличаются от полученных в предыдущих исследованиях для случая, когда предполагалось, что излучение полностью поглощается поверхностью капли. Отмечена возможность неустойчивости и множественности режимов равномерного движения.